M2 May 2012

1. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal*

A particle P moves in such a way that its velocity $\mathbf{v}$ m s^{-1} at time t seconds i

$$\mathbf{v} = (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}$$

(a) Find the magnitude of the acceleration of P when $t = 1$

(5)

Given that, when $t = 0$, the position vector of P is **i** metres,

(b) find the position vector of P when $t = 3$

(5)

a) $a = \frac{dv}{dt} = 6t\,i + (4-2t)j \quad t=1 \quad a = 6i + 2j$

$|a| = \sqrt{6^2 + 2^2} = \underline{6.32\,ms^{-2}}$ (3sf)

b) $S = \int v\,dt = (t^3 - t + c_1)i + (2t^2 - \frac{1}{3}t^3 + c_2)j$

$t=0 \quad i=1 \quad j=0 \qquad 0^3 - 0 + c_1 = 1 \quad \therefore c_1 = 1$

$2 \times 0^2 - \frac{1}{3}0^3 + c_2 = 0 \quad \therefore c_2 = 0$

$S = (t^3 - t + 1)i + (2t^2 - \frac{t^3}{3})j$

$t=3 \quad S = 25i + 9j$

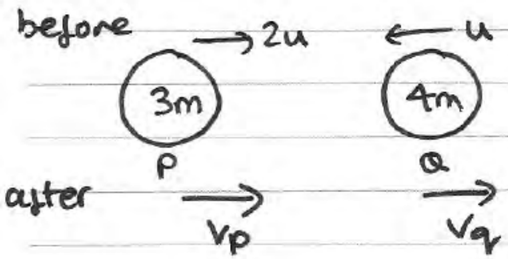
2. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on ... h bo... plane. The particle P collides directly with a particle Q of mass $4m$ mo... with speed u in the opposite direction to P. The coefficient of restitution be... is e.

(a) Find the speed of Q immediately after the collision.

Given that the direction of motion of P is reversed by the collision,

(b) find the range of possible values of e.

(5)



$$e = \frac{V_q - V_p}{3u} \qquad V_q - V_p = 3eu$$

$$\therefore \; V_p = V_q - 3eu$$

CLM $\quad 6\cancel{m}u - 4\cancel{m}u = 3\cancel{m}V_p + 4\cancel{m}V_q$

$$2u = 3V_q - 9eu + 4V_q \Rightarrow 7V_q = 2u + 9eu$$

$$\therefore \; V_q = \frac{1}{7}u(2+9e)$$

b) $V_p < 0 \Rightarrow \frac{2}{7}u + \frac{9}{7}eu - 3eu < 0$

$$\Rightarrow \frac{2}{\cancel{7}}\cancel{u} < \frac{12}{\cancel{7}}e\cancel{u} \Rightarrow 2 < 12e \Rightarrow e > \frac{1}{6}$$

$$\left(\therefore \frac{1}{6} < e \leq 1\right)$$

3.

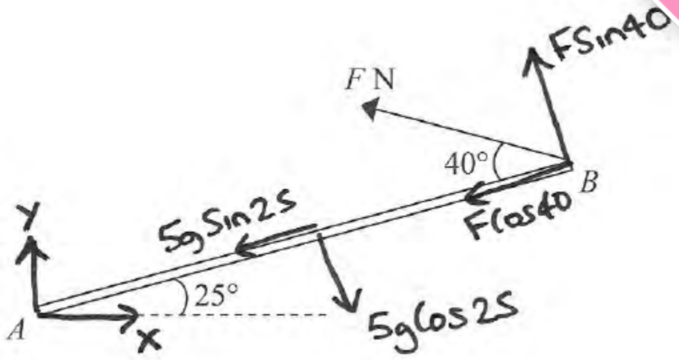


Figure 1

A uniform rod AB, of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

(a) Find the value of F.

(4)

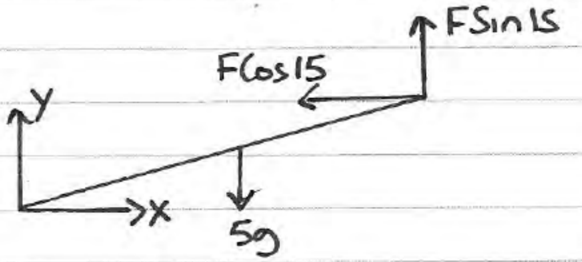
(b) Find the magnitude and direction of the vertical component of the force acting on the rod at A.

(4)

a) $A \circlearrowright \quad 5g\cos 25 \times 2 = F\sin 40 \times 4$

$\therefore \quad F = \dfrac{10g\cos 25}{4\sin 40} = 34.5\,\text{N}\ (3sf)$

b)



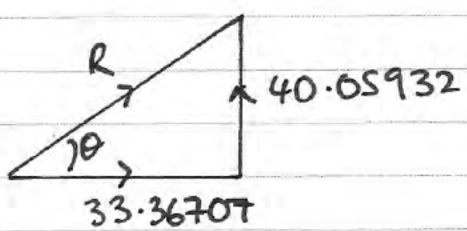
$RF\uparrow = 0$

$Y = 5g - F\sin 15$

$Y = 40.05932044$

$\overrightarrow{RF} = 0$

$X = F\cos 15 = 33.36707$



$R = \sqrt{33.36707^2 + 40.05932^2}$

$R = 52.1\,\text{N}\ (3sf)$

$\theta = \tan^{-1}\left(\dfrac{40.05932}{33.36707}\right)$

$\theta = 50.2°$ above horizontal

4.

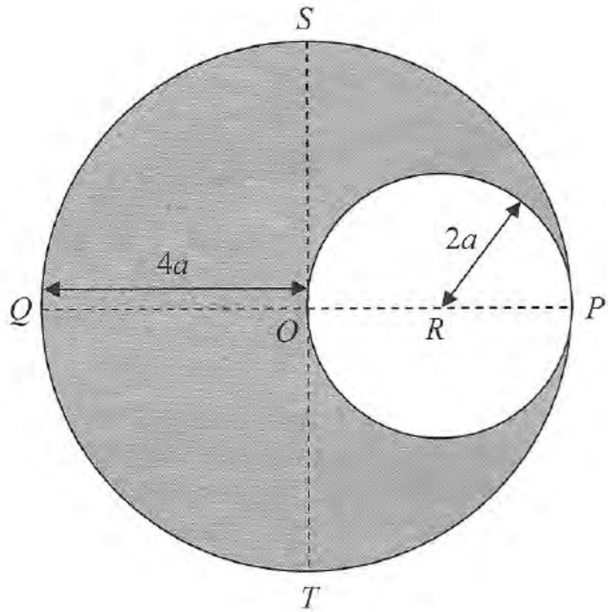


Figure 2

A uniform circular disc has centre O and radius $4a$. The lines PQ and ST are perpendicular diameters of the disc. A circular hole of radius $2a$ is made in the disc, with the centre of the hole at the point R on OP where $OR = 2a$, to form the lamina L, shown shaded in Figure 2.

(a) Show that the distance of the centre of mass of L from P is $\dfrac{14a}{3}$.

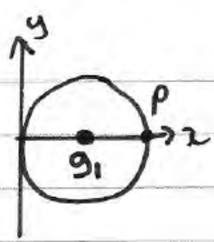
(4)

The mass of L is m and a particle of mass km is now fixed to L at the point P. The system is now suspended from the point S and hangs freely in equilibrium. The diameter ST makes an angle α with the downward vertical through S, where $\tan\alpha = \dfrac{5}{6}$.

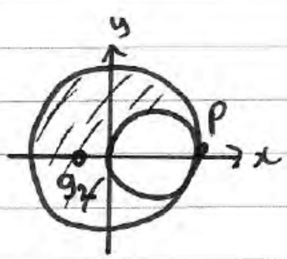
(b) Find the value of k.

mass per unit area $= t$

(5)

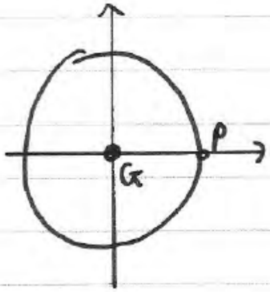


$g_1(2a, 0)$ $M = 4\pi a^2 t$



$g_2(\bar{x}, 0)$ $M = 16\pi a^2 t - 4\pi a^2 t$

$= 12\pi a^2 t$



$G(0,0) \quad M = 16\pi a^2 t$

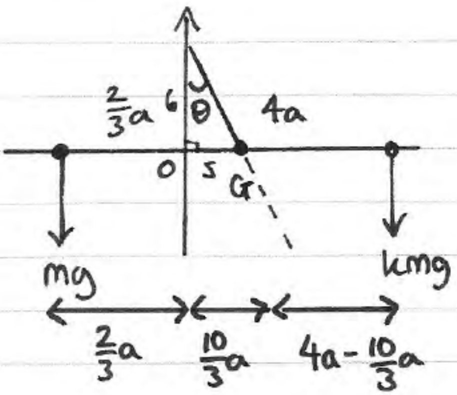
$$4\cancel{\pi} a^2 \cancel{t}\cancel{g} \times 2a + 12\cancel{\pi} a^2 \cancel{t}\cancel{g} \times \bar{x} = 16\cancel{\pi} a^2 \cancel{t}\cancel{g} \times 0$$

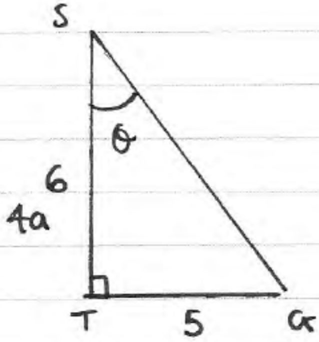
$$8a^{\cancel{3}} + 12\cancel{a^2}\,\bar{x} = 0 \Rightarrow 8a = -12\bar{x}$$

$$\therefore \bar{x} = -\frac{2}{3}a$$

x coordinate of g_2 from $P = 4a + \frac{2}{3}a = \frac{14}{3}a$ #

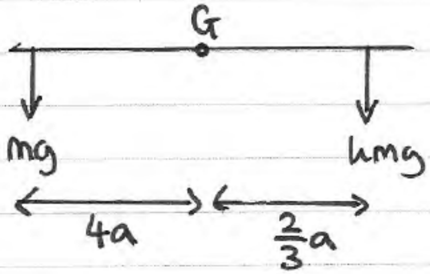
b)





$\Rightarrow \frac{5}{6}$ of $4a$

$\frac{10}{3}a$



G: $\cancel{mg} \times 4\cancel{a} = k\cancel{mg} \times \frac{2}{3}\cancel{a}$

$$4 = \frac{2}{3}k$$

$$\therefore \underline{k = 6}$$

5.

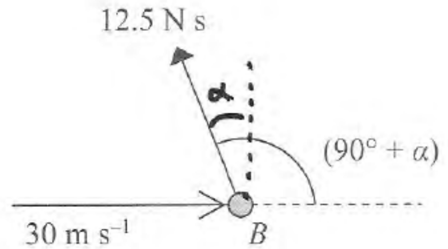


Figure 3

A small ball B of mass 0.25 kg is moving in a straight line with speed 30 m s^{-1} on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 N s and is applied in a horizontal direction making an angle of $(90° + \alpha)$, where $\tan\alpha = \frac{3}{4}$, with the initial direction of motion of the ball, as shown in Figure 3.

(i) Find the speed of B immediately after the impulse is applied.

(ii) Find the direction of motion of B immediately after the impulse is applied.

(6)

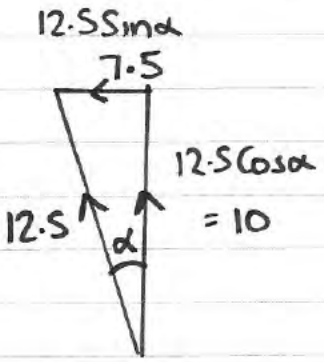
i) Momentum before $= 0.25 \times 30 = 7.5$ Ns

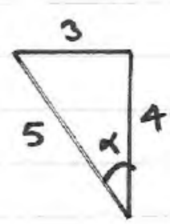
Impulse = change in momentum $= 12.5$

Momentum after $= 0.25v = -5$

$v = -20$ $\therefore$ speed $= \underline{20\text{ms}^{-1}}$

ii)





$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

Initial Mom $= 0.25\begin{pmatrix}30\\0\end{pmatrix} = \begin{pmatrix}7.5\\0\end{pmatrix}$

Impulse $= \begin{pmatrix}-7.5\\10\end{pmatrix}$

final mom $= \begin{pmatrix}0\\10\end{pmatrix} = 0.25v$ $\therefore$ $v = \begin{pmatrix}0\\40\end{pmatrix}$

change in mom = Impulse

speed $= 40\text{ms}^{-1}$ due North

6. A car of mass 1200 kg pulls a trailer of mass 400 kg up a straight road which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{14}$. The trailer is attached to the car by a light inextensible towbar which is parallel to the road. The car's engine works at a constant rate of 60 kW. The non-gravitational resistances to motion are constant and of magnitude 1000 N on the car and 200 N on the trailer.

At a given instant, the car is moving at 10 m s^{-1}. Find

(a) the acceleration of the car at this instant,

(5)

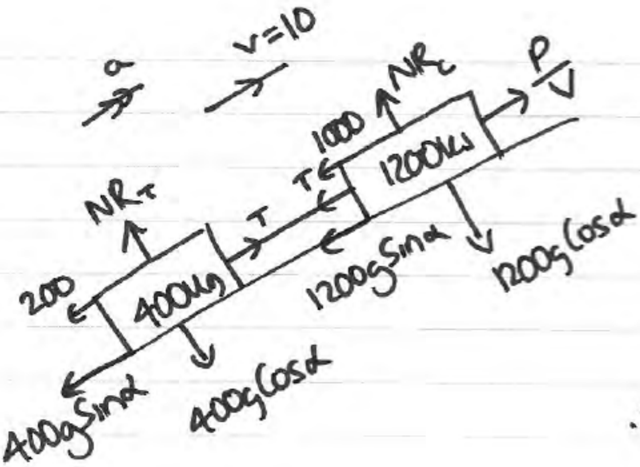
(b) the tension in the towbar at this instant.

(4)

The towbar breaks when the car is moving at 12 m s^{-1}.

(c) Find, using the work-energy principle, the further distance that the trailer travels before coming instantaneously to rest.

(5)



$\overrightarrow{RF} = ma$

$\frac{60000}{10} - 1000 - 200$

$-400g \times \frac{1}{14} - 1200g \times \frac{1}{14} = 1600a$

$\therefore\ 3680 = 1600a$

$a = \underline{2.3\,ms^{-2}}$

b) <u>trailer</u>

$\overrightarrow{RF} = ma \quad \Rightarrow \quad T - 200 - 400g \times \frac{1}{14} = 400 \times 2.3$

$T - 480 = 920$

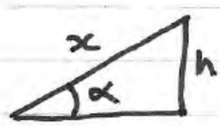
$\therefore\ \underline{T = 1400N}$

c) Initial KE = final PE + Wd against Res

$\frac{1}{2}(400) \times 12^2 = mg(\frac{1}{14}x) + 200 \times x$

$28800 = 280x + 200x = 480x$

$\therefore\ x = 60m.$



$\sin\alpha = \frac{1}{14} = \frac{h}{x}$

$h = \frac{1}{14}x$

7.

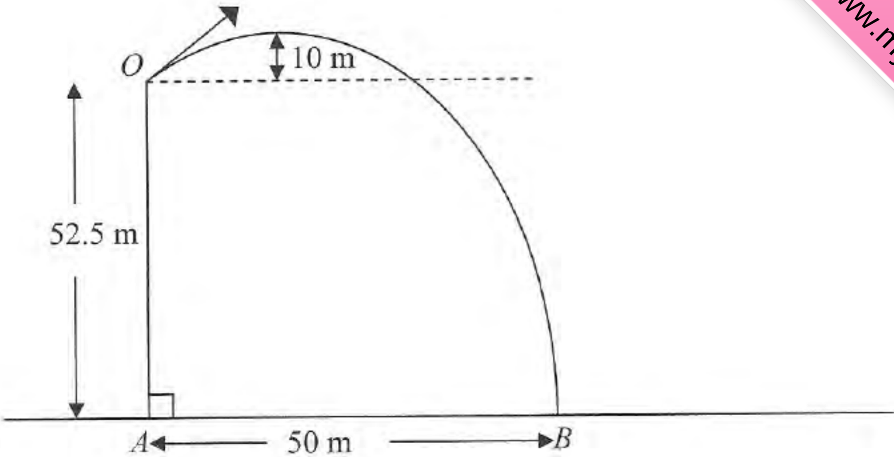


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}. **(3)**

(b) Find the speed of projection. **(9)**

(c) Find the time after projection when the stone is moving parallel to OB. **(5)**

a) $u\uparrow$ $\quad v^2 = u^2 + 2as$

$s\uparrow = 10$ $\quad 0 = u^2 - 19.6 \times 10$

$a\uparrow = -9.8$ $\quad \therefore u^2 = 196$

$v\uparrow = 0$ $\quad \therefore u = \underline{14} \text{ ms}^{-1}$

b) $u\uparrow = 14$ $\quad s = ut + \frac{1}{2}at^2$

$s\uparrow = -52.5$ $\quad -52.5 = 14t - 4.9t^2$

$a\uparrow = -9.8$

$49t^2 - 140t - 525 = 0$

$7t^2 - 20t - 75 = 0$

$(t-5)(7t+15) = 0$

$\therefore t = 5$

$\vec{H}$ $\quad$ Dist = vel × time

$$50 = \text{vel} \times 5 \quad \Rightarrow \quad V_h = 10$$

$V_h = 10$, $V_v = 14$ $\quad$ speed $= \sqrt{14^2 + 10^2} = 17\cdot$

(33

$$OB = \begin{pmatrix} 50 \\ -52.5 \end{pmatrix}$$

$$\text{vel} = k\begin{pmatrix} 50 \\ -52.5 \end{pmatrix} = \begin{pmatrix} 50k \\ -52.5k \end{pmatrix}$$

$V_h = 10 \quad \therefore k = \frac{1}{5} \quad \Rightarrow \quad V_v = -10.5$

$u\uparrow = 14$ $\qquad$ $v = u + at$

$a\uparrow = -9.8$ $\qquad$ $-10.5 = 14 - 9.8t$

$v\uparrow = -10.5$ $\qquad$ $-24.5 = -9.8t \quad \therefore t = 2.5\text{sec}$